Birzeit University

Mathematics Department

Math 234

97

First Hour Exam

First semester 2013/2014

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Question #1 (24%): Which of the following statements is true and which is false:

false 1- If A is an nxn matrix and the system AX=0 has a nontrivial solution then A is nonsingular

True 2- If A and B are nxn matrices and AB is nonsingular then both A and B are nonsingular.

false .3-If A is 4x4 matrix then $|-A| = -|A|$

~~False~~ 4- If A is 3x3 matrix and $A = -A^T$ then A is singular $|A| = 0$

false 5- The product of two symmetric nxn matrices is symmetric .

True 6-If A is a nonsingular matrix then the matrix A^T is nonsingular also.

false 7- Any non homogenous system of linear equations that has a nontrivial solution must have infinite number of solutions.

False 8- If A, B, C are 2x2 matrices with AB=AC then B=C.

False 9- If A and B are 2x2 matrices such that A.B=0 then A=0 or B=0.

21

True 10- If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$ then $A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$

True 11- If A, B are 3x3 matrices with $|A| = 4$, $|B| = 5$ then $|2A^{-1}B| = 10$

$8 \quad \frac{1}{4} \quad 5$

~~True~~ 12- If $A = \begin{pmatrix} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{pmatrix}$ then the (2, 3) entry of A^{-1} is $\frac{1}{3}$

$2 \; 5 \quad 2 \; 1 \quad 36+40+7 \quad 6-2 \quad -(42+8+30)$

$$A^{-1}_{32} = \frac{A_{\cancel{32}}}{|A|} = \frac{1}{3}$$

True 13-If the coefficient matrix of the system AX=b is $A=\begin{pmatrix}1 & 2 & 3\\ 0 & -1 & 1\\ 0 & 4 & 1\end{pmatrix}$

Then the system must have a unique solution

True 14- Any nonsingular matrix can be written as a product of elementary matrices.

False 15-The product of two elementary matrices is elementary

True 16- $|AB|=|BA|$ for any two nxn matrices A and B

$|A|\,|B| = |B|\,|A|$

Question #2(30%): Circle the correct answer:

1– If $A=\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$, and $|A|=6$, and $B=\begin{bmatrix} 2a & 2b & 2c\\ 4g & 4h & 4i\\ -d & -e & -f\end{bmatrix}$ then $|B|=$

$=8\,|A| = 8\cdot 6 = 48$

a) 48   b) -48

$-1 * 2 * -1 * 4$

c) -24 d)24

2- If A, B are two $n\times n$ matrices then

a) $\det(AB^T)=\det(AB) \rightarrow$ $|A|\,|B^T| = |A|\,|B|$ ✓.

b) $\det(\alpha A)=\alpha\det(A)$ x $\alpha^n|A|$

c) $\det(A+B)=\det(A)+\det(B)$ X

n_1 n_2 n_3 nn

3– If AX=b has no solution where A is an nxn matrix and b is an nx1 matrix then :

a- A is row equvalent to I_n.

b-A is nonsingular .

c- A is a product of elementary matrices. → $I \sim A$ nonsingular

d- $AX=0$ has infinitely many solutions,

4– The conditions on a,b such that the system $\begin{aligned} ax + y &= 1\\ 2x + y &= b\end{aligned}$ has infinite number of solution is

a) a=2 and b=1

b) $a\neq 2$ and b=1

c) a=2 and $b\neq 1$

d) $a\neq 2$ and $b\neq 1$

$\left[\begin{array}{cc|c} a/a & 1/a & 1/a\\ 2 & 1 & b\end{array}\right] \rightarrow \left[\begin{array}{cc|c} 2/2 & 1/2 & b/2\\ a & 1 & 1\end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 1/2 & b/\\ 0 & 1-\frac{a}{2} & 1-\end{array}\right]$

5–If A and B are $n\times n$ nonsingular matrices then :

a- $(AB)^T=A^TB^T$ X B^TA^T

b- $(A+B)^T=A^T+B^T$

c- $(AB)^{-1}=A^{-1}B^{-1}$ X $B^{-1}A^{-1}$

d- $|\alpha A|=\alpha^n|A|$ X

$1-\frac{a}{2}=0$ $1-\frac{ab}{2}$;

$1=\frac{a}{2}$ $1=\frac{ab}{2}$

$a=2$ $2=a*b$

$a=2$, $b=1$

6 – If A is an nxn nonsingular matrix then one of the following is false :

a) A is row equavalent to the identity matrix I_n

b) $AX = b$ has a unique solution for every nx1 matrix b

c) A is a product of elementary matrices

d) $AX = 0$ has a nontrivial solution .

7- Asystem of linear equations AX=b consisting of two equations in four unknowns has

$\overset{2}{m} < \overset{4}{n}$

a) a unique solution

b) Infinite number of solutions

c) No solution.

d) Infinite number of solutions or no solution.

8 – the values of a for which the matrix A= $\begin{pmatrix} 2 & 1 & 1 \\ 0 & a & 5 \\ a & 0 & 0 \end{pmatrix}$ is singular is :

a) {0,5} b) {0,2}

c) {2,5} d) {5}

$a \begin{vmatrix} 1 & 1 \\ a & 5 \end{vmatrix} = a(5-a) = 0$

$a = 0 , a = 5 .$

9 – If A is 3x3 matrix such that A . $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ then

a) A is singular. b) A is nonsingular .

c) A is the zero matrix . d) the system AX=0 has only one solution

10 – One of the following matrices is in the row echelon form but not in reduced row echelon form

a) $\begin{bmatrix} 1 & 0 & 7 & 0 \\ 0 & 2 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 3 & 1 \\ 0 & 2 & 5 \\ 1 & 6 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 2 & 1 & 5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\det(A^{-1}) = \det(A)^{-1}$

Q1 (a) (12%) Use Gauss Jordan reduction to solve the following system of linear equations

$$x_1 + 2x_2 - x_3 + x_4 = 6$$
$$2x_1 + x_2 - 2x_3 + 5x_4 = 12$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 6 \\ 2 & 1 & -2 & 5 & 12 \end{array}\right]$$

$$-2R_1+R_2 \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 6 \\ 0 & -3 & 0 & 3 & 0 \end{array}\right]$$

$$-\frac{1}{3}R_2 \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 6 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$-2R_2+R_1 \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 6 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

Free variables :- $X_3 = \alpha$, $X_4 = \beta$

leading ones :- X_1 , X_2.

$X = (\alpha - 3\beta + 6, \beta, \alpha, \beta)$.

2

$$X_1 - X_3 + 3X_4 = 6$$
$$X_1 = X_3 - 3X_4 + 6$$
$$X_1 = \alpha - 3\beta + 6$$

$$X_2 - X_4 = 0$$
$$X_2 = X_4$$
$$X_2 = \beta$$

b)(8%) Let A and B be symmetric matrices and suppose also that AB is symmetric Show that AB=BA

$A^T = A$, $B^T = B$.

$(AB)^T = AB$.

$(AB)^T = B^T A^T = BA \rightarrow$ (A symmetric and so as B.)

and $(AB)^T = AB$ « symmetric » (AB).

so

$AB = BA$

20

Q3 (12%)(a) **Consider the following system of linear equation**

Find conditions on a, b such that the system

1) has one solution unique → m=n
2) has infinite number of solutions
3) has no solution

$$\begin{aligned} x_1 \quad\quad\quad + 2x_3 &= 1 \\ -x_1 + x_2 - x_3 &= 0 \\ -x_1 \quad\quad\quad + ax_3 &= b \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & a & b \end{array}\right]$$

$$\begin{array}{c} \\ R_1+R_2 \\ R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & (2+a) & (1+b) \end{array}\right]$$

12

① one solution a unique solution.

$2+a \neq 0$

$a+2=0$, $a \neq -2$, $b = (-\infty, \infty)$

② infinite number of solutions.

$2+a=0$ and $1+b=0$

$a=-2$, $b=-1$

③ no solution:-

$2+a=0$, $b+1 \neq 0$

$a=-2$, $b=(-\infty, \infty)-\{-1\}$, $(b \neq -1)$.

$$\begin{array}{r} 14 \\ 14+ \\ 14+ \\ \hline 42 \end{array} \qquad 14+14+14 \qquad 32$$

Question 3 (14%) a) Let A be a 3×3 nonsingular matrix . If adj A= $\begin{pmatrix} 2 & 1 & -2 \\ -7 & 4 & 2 \\ 4 & -3 & 1 \end{pmatrix}$

$= (8+8+-42)-(-32-\ldots$
$= 16-42+51$
$= 16+9 = 25$

a)(3points) Find |adj(A)|

$$|\text{adj}A| = 2\begin{vmatrix} 4 & 2 \\ -3 & 1 \end{vmatrix} - \begin{vmatrix} -7 & 2 \\ 4 & 1 \end{vmatrix} + -2\begin{vmatrix} -7 & 4 \\ 4 & -3 \end{vmatrix}$$
$$= 2(4+6) - (-7-8) - 2(21-16)$$
$$= 20 + 15 - 10 = 25$$

3

b) (6points) Find adj(adj(A))

$$\text{adj adj}A = \begin{pmatrix} \begin{vmatrix} 4 & 2 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} -7 & 2 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} -7 & 4 \\ 4 & -3 \end{vmatrix} \\ -\begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -2 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix} \\ \begin{vmatrix} 1 & -2 \\ 4 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -2 \\ -7 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -7 & 4 \end{vmatrix} \end{pmatrix}^T$$

6

$$= \begin{pmatrix} 10 & 15 & 5 \\ 5 & 10 & 10 \\ 10 & 10 & 15 \end{pmatrix}^T = \begin{pmatrix} 10 & 5 & 10 \\ 15 & 10 & 10 \\ 5 & 10 & 15 \end{pmatrix}$$

c)(5points) Find the matrix A

$$|A| = \sqrt[n-1]{|\text{adj}A|} = \sqrt[2]{25} = 5$$

$$\frac{1}{|A|}\text{adj adj}A = A.$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A = \begin{pmatrix} 2/5 & 1/5 & -2/5 \\ -7/5 & 4/5 & 2/5 \\ 4/5 & -3/5 & 1/5 \end{pmatrix}$$

$$(A^{-1})^{-1} = A$$

$$A = \begin{pmatrix} 2/5 & 1/5 & -2/5 \\ -7/5 & 4/5 & 2/5 \\ 4/5 & -3/5 & 1/5 \end{pmatrix}^{-1}$$

$\rightarrow$ Cont

14